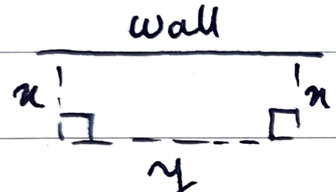
No. Date. June 30, 2026

Today's pun: The one L lama is a priest,
the two L llama is a beast,
but I'll bet my silly pyjama that there's
no three L lllama! Ogden Nash

More optimization – Curve sketching



9 m of fencing
→ How big a rectangular area can we fence off against the wall?

$2x + y = 9 \Rightarrow 2x = 9 - y \Rightarrow x = \frac{9-y}{2}$
or
$y = 9 - 2x$

$A = \text{Area} = xy$

$\Rightarrow A = xy = \left(\frac{9-y}{2}\right)y = \frac{9y - y^2}{2}$

$\frac{dA}{dy} = \frac{9}{2} - y = 0 \iff y = \frac{9}{2} = 4.5$

Candidates for maximum area:

$\frac{dA}{dy} = 0 \Rightarrow y = \frac{9}{2}$ & $x = \frac{9}{4} = 2.25$

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$A = \frac{81}{4} - \frac{81}{8} = \frac{81}{8} > 0$

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what're the possible values of y?

$$0 \leq y \leq 9$$

if $y = 0 \Rightarrow A = x(0) = 0$

if $y = 9 \Rightarrow A = 0$

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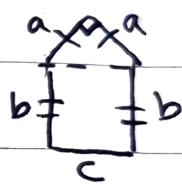
another example:

we have $8m^2$ of glass for a window which we can get in any shape we like.



→ we want an isosceles triangle over a rectangle for the shape.

what is the minimum perimeter of such a window?

① $A = \frac{a^2}{2} + bc = 8$

$P = 2a + 2b + c$

Pythagoras:

$c^2 = a^2 + a^2 = 2a^2$

$\Rightarrow c = \sqrt{2}a$

② $A = \frac{a^2}{2} + b \cdot \sqrt{2}a \quad \Rightarrow b = \frac{8 - a^2}{\sqrt{2}a} = \frac{4\sqrt{2}}{a} - \frac{a}{\sqrt{2}}$

$P = 2a + 2\left(\frac{4\sqrt{2}}{a} - \frac{a}{\sqrt{2}}\right) + \sqrt{2}a$ ... → Next page!

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$$\cdots = 2a + 2\left(\frac{4\sqrt{2}}{a} - \frac{a}{\sqrt{2}}\right) + \sqrt{2}a \Rightarrow$$

$$2a + \frac{8\sqrt{2}}{a} - \cancel{\frac{2a}{\sqrt{2}}} + \cancel{\sqrt{2}a} \Rightarrow$$

$$\frac{dP}{da} = \frac{d}{da}\left(2a + \frac{8\sqrt{2}}{a}\right) \Rightarrow 2 + 8\sqrt{2}(-1a^{-2}) \Rightarrow$$

$$2 - 8\sqrt{2}\,a^{-2} \Rightarrow$$

$$2 - \frac{8\sqrt{2}}{a^2} = 0 \Rightarrow$$

$$\frac{8\sqrt{2}}{a^2} = 2 \Rightarrow 8\sqrt{2} = 2a^2 \Rightarrow a^2 = \frac{8\sqrt{2}}{2} \Rightarrow$$

$$a = \pm\sqrt{4\sqrt{2}} \Rightarrow a = 2\sqrt{\sqrt{2}} = 2\sqrt[4]{2} = 2\cdot 2^{\frac{1}{4}} = \boxed{2^{\frac{5}{4}}}$$

$$0 \leq a \leq 4$$

$$\hookrightarrow b = 0 \Rightarrow \frac{a^2}{2} = 8 \Rightarrow a^2 = 16$$

$$\boxed{a = 4}$$

$$\begin{cases} a = 0 & A = 0 \\ a = 4 & A = \end{cases}$$

$$a = 4 \Rightarrow b = \sqrt{2} - 2\sqrt{2} = -\sqrt{2}$$

Candidates P

$a = 2^{\frac{5}{4}}$ $\quad 2.2^{5/4} + \frac{8}{\sqrt{2}}.2^{5/4} = 4.2^{\frac{1}{4}} + 8.2^{\frac{3}{4}} \approx 18.2$

$\boxed{2a + \frac{8\sqrt{2}}{a}}$ use this

$a = 0$ $\quad \infty$

$a = 4$ $\quad 8 + 2\sqrt{2} \approx 10.8$

Qualitative Analysis

* Find out what you can about the graph before drawing it.

$y = xe^{-x}$

1° Domain (where is it defined?)

All $x \in \mathbb{R}$, $(-\infty, \infty)$

2° Intercepts

y-int (set $x=0$): $y = 0e^{-0} = 0 \cdot 1 = 0$
x-int (set $y=0$): $xe^{-x} = 0 \Rightarrow x = 0$

$\left.\begin{array}{l} \\ \end{array}\right\} (0,0)$

3° Vertical Asymptotes

$y = xe^{-x}$ is defined for all x & continuous for all x where it is defined.

so no VA because these're discontinuities.

4° Horizontal Asymptotes

we look how the functions behave as we go to $+\infty$ & $-\infty$

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$$\lim_{x\to-\infty} xe^{-x} = -\infty$$

$$\lim_{x\to\infty} ex^{-x} = 0 \quad \text{because} \quad \lim_{x\to\infty} \frac{x^{\to\infty}}{e^{x}_{\to\infty}} = \lim_{x\to\infty} \frac{\frac{d}{dx}x}{\frac{d}{dx}e^{x}} \Rightarrow$$

$$\lim_{x\to\infty} \frac{1^{\to 1}}{e^{x}_{\to\infty}} = 0$$

Thus we have a one way horizantal asymptote $y=0$ in the positive direction.

5° Max / Min / increasing / decreasing

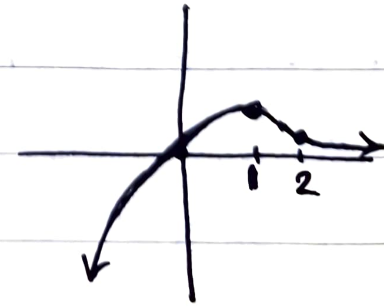
$$\frac{dy}{dx} = \frac{d}{dx} xe^{-x} = \left(\frac{d}{dx}x\right)e^{-x} + x\left(\frac{d}{dx}e^{-x}\right) \Rightarrow$$

$$1 \cdot e^{-x} + x \cdot e^{-x} \cdot \left(\frac{d}{dx}(-x)\right) \Rightarrow$$

$$1 \cdot e^{-x} + xe^{-x}(-1) = (1-x)e^{-x}$$

$$\frac{dy}{dx} = (1-x)e^{-x} \begin{cases} >0 & \text{if } 1-x>0 \quad 1>x \\ =0 & \text{if } 1-x=0 \quad x=1 \\ <0 & \text{if } 1-x<0 \quad 1<x \end{cases}$$

x	$(-\infty, 1)$	1	$(1, \infty)$
$\frac{dy}{dx}$	+	0	−
y	↑	max	↓



6° Concavity & Inflection

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left[(1-x)e^{-x}\right] \Rightarrow$$

$$\left[\frac{d}{dx}(1-x)\right]e^{-x} + (1-x)\left[\frac{d}{dx}e^{-x}\right] \Rightarrow (x-2)e^{-x}$$

$$\frac{d^2y}{dx^2} = (x-2)e^{-x} \quad \begin{cases} x > 2 \\ x = 2 \\ x < 2 \end{cases}$$

x	$(-\infty, 2)$	2	$(2, \infty)$
$\frac{d^2y}{dx^2}$	$-$	0	$+$
y	$\cap$		$\cup$

inflection point